

压气机叶片排中三维流场计算的多网格法

西北工业大学 宁卫** 周新海

【摘要】 以三维有限差分形式的 MacCormack 格式为基础,对相对圆柱坐标系下弱守恒型控制方程,导出了残差在粗、细网格上的传播与回插公式,形成了一种多网格算法。计算表明,这种算法大幅度地提高了计算收敛速度,而且方法本身简便易行。

关键词: 三维流动 压气机 有限差分法

分类号: V231.3

1 前言

时间相关显式方法由于程序结构简单,而且可获得很高的向量化程度,因此成为叶栅三维流场计算的重要算法。但是显式方法是条件稳定的,需要的计算时间长,为了加速收敛,采用多网格技术是大幅度提高计算效率的重要途径。在叶栅流场计算中得到广泛应用的多网格法是 Ni 格式^[1],它是有限体积法为基础的。本文的工作是:发展一种非有限体积的多网格算法,在第一层网格上本文没有采用 Ni 格式,而是采用已有工作基础的有限差分形式的 MacCormack 格式^[2],由此出发,借助 Ni 的残差分配原理,导出了三维相对圆柱坐标系下多网格公式。还讨论了加速算法中控制方程中源项的影响。所形成的三维 Euler 方程多网格解法通过实际计算证明,它大幅度提高了计算收敛速度,可望在跨音压气机全三维流场校核计算中发挥作用。

2 控制方程和基本算法

将相对圆柱坐标系 (r, θ, z) 下的三维 Euler 方程转换到一般曲线坐标系 (ξ, η, ζ) 上,可写成如下形式:

$$\partial Q / \partial \alpha + \partial F / \partial \xi + \partial G / \partial \eta + \partial H / \partial \zeta = K \quad (1)$$

式中 Q 是原参数, F, G, H 是通量, K 是源项,具体表达式可参见文献[2]。

在细网格点上(网点间距为 h 的各点),求解方程(1)采用 MacCormack 显式两步格式:

$$Q_{i,j,k}^* = Q_{i,j,k}^n - (\Delta t / \Delta \xi) (F_{i,j,k}^n - F_{i-1,j,k}^n) - (\Delta t / \Delta \eta) (G_{i,j,k}^n - G_{i,j-1,k}^n) - (\Delta t / \Delta \zeta) (H_{i,j,k}^n - H_{i,j,k-1}^n) + \Delta t K_{i,j,k}^n \quad (2)$$

$$Q_{i,j,k}^{**} = (1/2) \{ \bar{Q}_{i,j,k}^* + Q_{i,j,k}^n - (\Delta t / \Delta \xi) (F_{i+1,j,k}^* - F_{i,j,k}^*) - (\Delta t / \Delta \eta) (G_{i,j+1,k}^* - G_{i,j,k}^*) - (\Delta t / \Delta \zeta) (H_{i,j,k+1}^* - H_{i,j,k}^*) + \Delta t K_{i,j,k}^* \} \quad (3)$$

$Q_{i,j,k}^{**}$ 是基本格式求出的 $(n+1)\Delta t$ 时刻的解。细网格点上参数变化量为 $\Delta Q_{i,j,k}^h = Q_{i,j,k}^{**} - Q_{i,j,k}^n$ 。

3 多网格算法

算法包括两个步骤,首先在 h 网格上用基本格式得到参数变化量 $\Delta Q_{i,j,k}^h$,在 $2h$ 粗网格(网

点是距为 $2h$ 的网格) 上的变化量 $\delta Q_{i,j,k}^{2h}$ 由下述分配公式给出:

$$\begin{aligned} \delta Q_{i,j,k}^{2h} = & \frac{1}{8} \{ \triangle Q - (\triangle t / \triangle \xi) \triangle F - (\triangle t / \triangle \eta) \triangle G - (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i+1,j+1,k+1}^h \\ & + \triangle Q + (\triangle t / \triangle \xi) \triangle F - (\triangle t / \triangle \eta) \triangle G - (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i-1,j+1,k+1}^h \\ & + \triangle Q - (\triangle t / \triangle \xi) \triangle F + (\triangle t / \triangle \eta) \triangle G - (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i+1,j-1,k+1}^h \\ & + \triangle Q + (\triangle t / \triangle \xi) \triangle F + (\triangle t / \triangle \eta) \triangle G - (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i-1,j-1,k+1}^h \\ & + \triangle Q + (\triangle t / \triangle \xi) \triangle F - (\triangle t / \triangle \eta) \triangle G + (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i+1,j+1,k-1}^h \\ & + \triangle Q - (\triangle t / \triangle \xi) \triangle F + (\triangle t / \triangle \eta) \triangle G + (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i+1,j-1,k-1}^h \\ & + \triangle Q - (\triangle t / \triangle \xi) \triangle F - (\triangle t / \triangle \eta) \triangle G + (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i+1,j+1,k-1}^h \\ & + \triangle Q + (\triangle t / \triangle \xi) \triangle F + (\triangle t / \triangle \eta) \triangle G + (\triangle t / \triangle \zeta) \triangle H + (\triangle t / 2) \triangle K \}_{i-1,j-1,k-1}^h \end{aligned} \quad (4)$$

式中 $\triangle F = (\partial F / \partial Q) \triangle Q$, $\triangle G = (\partial G / \partial Q) \triangle Q$, $\triangle H = (\partial H / \partial Q) \triangle Q$, $\triangle K = (\partial K / \partial Q) \triangle Q$ 。

将 $\delta Q_{i,j,k}^{2h}$ 用线性插值回插到 h 网格上, 即 $\delta Q_{i,j,k}^h = I_{2h} \delta Q_{i,j,k}^{2h}$, 于是一个时间步长后细网格点上的参数为 $Q_{i,j,k}^{n+1} = Q_{i,j,k}^{*} + \delta Q_{i,j,k}^h$, 由此, 网格还可以逐步变粗, 实现在 $4h, 8h, \dots$ 网格上的分配与回插计算。本文在求跨音速压气机转子叶排的三维 *Euler* 解中仅采用了两重网格。值得注意的是: (4) 式中包含的 $\triangle K$ 代表源项的影响。本文在加速算法中对 $\triangle K \neq 0$ 和 $\triangle K = 0$ 两种方案作了比较, 发现最终的收敛解和收敛步数完全相同, 这说明变化量在粗网格上的传播中, 通量项起主导作用, 弱守恒型方程的源项在分配公式中可以略去。

4 计算结果及分析

算例 1 某压气机第 1 级转子叶片, 是一个高负荷跨音速转子。计算网格取 $13 \times 13 \times 65$ 。

图 1 是利用多网格法计算得到的吸力面及压力面上相对马赫数等值图, 图 2 是叶尖、叶中、叶根处叶片表面相对马赫数分布, 可以看出, 流场中激波的分辨较好, 参数分布合理。此外, 还用基本格式进行了计算, 得到了与上述计算相同的结果。图 3 是两者的收敛史, 可见, 达到相同的收敛标准, 多网格法的收敛步数仅为单网格的 $1/3$ 。

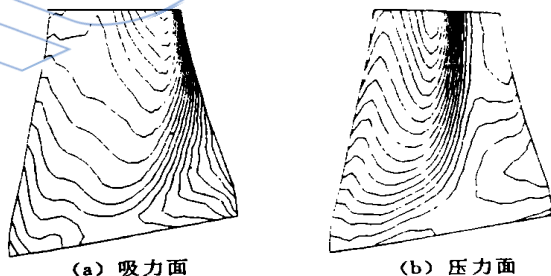


图 1 叶片表面上相对等马赫数等值线

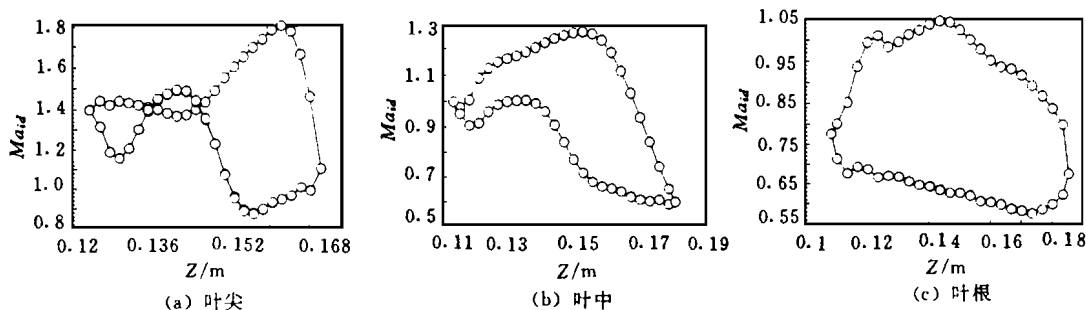


图 2 叶片表面上相对等马赫数分布

算例 2 某型低压压气机第 1 级转子叶片。利用多网格法计算了转子叶排中的跨音速三维流场, 图 4 是用基本格式和多网格计算的收敛史。可以看出, 基本格式的残差在后期出现水平振荡, 而多网格法则有效地克服此类问题。

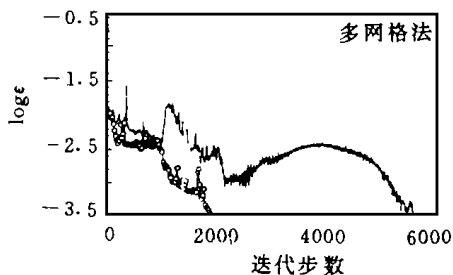


图3 算例 1 的收敛史

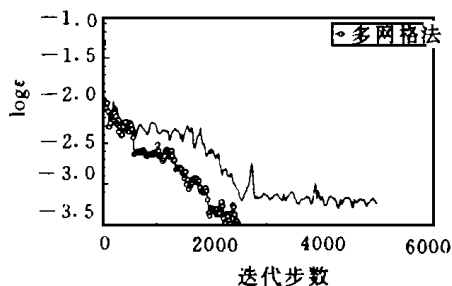


图4 算例 2 的收敛史

本法的数值实践表明, 在跨音速压气机回转叶片排的三维流场计算中, 由于其残差的波动较大, 因此, 采用多网格, 即使是 $h \rightarrow 2h$ 两重网格算法也可以取得十分显著的加速收敛效果。

参 考 文 献

- 1 Ni R H. A Multiple-Grid Scheme for Solving the Euler Equations. AIAA J. 1982, 20(1)
- 2 宁卫, 周新海等. 跨音速压气机回转叶片排中三维流场的数值分析. 工程热物理学报, 1991, 12(1)

(责任编辑 魏星禄)

TECHNICAL NOTES

A MULTIGRID SCHEME FOR CALCULATING THREE-DIMENSIONAL FLOWS THROUGH COMPRESSOR BLADE ROW

Ning Wei and Zhou Xinhai

(*Aeroengine Department, Northwestern Polytechnical University Xian 710072*)

ABSTRACT

Based on the MacCormack explicit difference Scheme, a multigrid scheme has been developed to solve the three-dimensional Euler equations expressed by weak conservation law in cylindrical coordinate system. The distribution and interpolation formulas of corrections for the presented multigrid scheme have been derived. The present Euler solver was used to calculate the flows of two sets of transonic compressor rotor blade rows, and proven to be a valid and useful design tool. The computed results indicate that this scheme is enable to accelerate the computation convergence greatly, and the method is accurate and reliable.

Key words Compressors Three-dimensional flow Finite difference methods

DESIGN AND EXPERIMENT OF CENTRIFUGAL COMPRESSOR 3-D IMPELLERS FOR HEAVY GASES

Wang Shangjin, Xi Guang, Liu Sixue

(*Dept. of Power Machinery, Xi'an Jiaotong University, Xi'an, 710049*)

Yuan Yan, Wang Siyi

(*Chong Qing Machinery Manufacturing Factory, Chong Qing*)

ABSTRACT

An inverse problem method based on s2 flow surface theory of 3-D flow field has been developed to design centrifugal compressor impellers for heavy gases and some techniques to improve the performance of high pressure ratio impellers have been presented too. Two impellers with Mach number $M_{u_2} = 1.26, 1.06$ and relative width $b_2/D_2 = 0.054, 0.036$ respectively have been designed by use of this method. Experiments of the impellers have been performed in a closed-loop test rig with Freon gases (R11) to verify the design method. The dynamic pressures distribution from shroud to hub at the exits of impellers and the losses of vaneless diffusers and volutes have been measured and analyzed under different work points. Their good performance obtained from tests shows the design method and techniques presented in this paper is feasible and favourable.

Key words Centrifugal compressor 3-D impeller Design and experiment

A MODEL FOR ESTIMATING TIP CLEARANCES IN GAS TURBINE ENGINE TRANSIENTS

Zhu Zhili and Liao Kuo

(*4th Dept. Beijing University of Aeronautics and Astronautics, Beijing 100083*)

ABSTRACT

During the gas turbine engine transient, the clearances of the compressor and turbine blade tips differ from those in stable operation because the heat change between gas and components causes the different radial movements of the rotor and case. In order to simulate the real thermodynamics process in the gas turbine engine transient as possible, a model for estimating the variations of the tip clearance and efficiency with time is given. In this model, the typical stage chosen from a multi-stage turbomachinery will be decomposed into three components (case, blade and disc), the instantaneous heat growth of each component is calculated individually by means of the different heat transfer model, and the instantaneous centrifugal growths of blade and disc are also estimated, therefore the instantaneous tip clearance are obtained by summing the heat and centrifugal growths of all compo-